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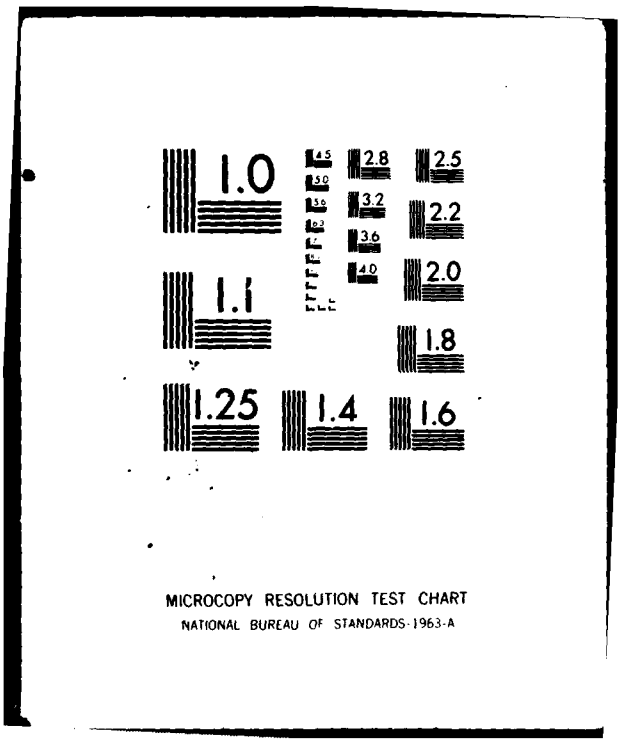
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APPROXIMATION TECHNIQUES FOR PARAMETER ESTIMATION IN HEREDITARY CONTROL SYSTEMS*

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APPROXIMATION TECHNIQUES FOR PARAMETER ESTIMATION IN HEREDITARY CONTROL SYSTEMS

H. T. Banks and I. G. Rosen

Abstract

Two types of approximation techniques for parameter estimation of delay systems are described and compared. One involves state discretization only while the other entails simultaneous state and time discretization.

We consider two approximation techniques for parameter identification problems for delay systems of the form

$$\dot{x}(t) = Lx_t + Bu(t) \qquad 0 \leq t \leq T \tag{1}$$

$$x(0) = \eta \qquad x_0 = \phi$$

where B is an $n \times m$ matrix, u is an R^m valued function that is piecewise continuous on $[0,T]$ (i.e., $u \in PC(0,T)$), $\eta \in R^n$ and ϕ is an R^n valued function that is square integrable

on $(-r,0)$ (i.e., $\phi \in L_2^n(-r,0)$). For $x:[-r,T] \to R^n$ measurable and $t \in [0,T]$, we denote by x_t the measurable function given by $x_t(\theta) = x(t+\theta)$, $-r \leq \theta \leq 0$. The linear operator $L:L_2^n(-r,0) \to R^n$ is assumed to be of the form:

$$L\phi = \sum_{j=0}^{\nu} A_j \phi(-\tau_j) + \int_{-r}^{0} A(s)\phi(s)ds \tag{2}$$

where $0 = \tau_0 < \tau_1 < \tau_2 \ldots < \tau_\nu = r$, A_j, $j = 0,1,2\ldots\nu$, are $n \times n$ matrices and A is an $n \times n$ matrix valued function which is square integrable on $(-r,0)$. Strictly speaking the expression for L given by (2) is not well defined for all $\phi \in L_2^n(-r,0)$ in that point evaluations of ϕ are required. However, with the usual interpretation (see [7]) the system (1) has well defined solutions for all $(\eta,\phi) \in R^n \times L_2^n(-r,0)$.

The basic parameter identification problem (PID) is one of fitting to data a model such as (1) (where u is a fixed input, $A_i = A_i(q)$ $i = 0,1,2\ldots\nu$, $A(\cdot) = A(\cdot,q)$ and $B = B(q)$ are continuous functions of some parameter $q \in R^K$) by choosing parameter values q from some compact set $\mathcal{Q} \subset R^K$ and initial conditions η,ϕ from some compact $\mathcal{S}$ contained in $R^n \times L_2^n(-r,0)$. To be more specific, assume that one is given $\xi \in C^n(0,T)$ ($C^n(0,T)$ denoting the collection of R^n valued continuous functions defined on $(0,T)$) that represents measurements on $[0,T]$ of the "observables" $c(t,q) = cx(t;q) + du(t)$ for (1) where $t \to x(t;q)$ is the solution to (1) corresponding to a value $q \in \mathcal{Q}$. (If, as usual, only dis-

crete measurements are made, the curve ξ may be constructed via some interpolation scheme.) The problem then is to find a value $\bar{\gamma} = (\bar{q},(\bar{\eta},\bar{\phi})) \in \mathcal{Q} \times \mathcal{S}$ such that

$$J(\gamma) = |c(0,q) - \xi(0)|^2_{W_1} + |c(T,q) - \xi(T)|^2_{W_2}$$

$$+ \int_0^T |c(s,q) - \xi(s)|^2_{W_3} ds$$

attains a minimum on $\mathcal{Q} \times \mathcal{S}$ at $\gamma = \bar{\gamma}$. Here W_1, W_2, W_3 represent positive definite weighting matrices.

These problems are infinite dimensional state system problems and our approach involves first rewriting (1) as an equivalent ordinary differential equation (ODE) in an appropriately chosen Hilbert space Z. As the state space we choose $Z = R^n \times L_2^n(-r,0)$ since one can argue equivalence of (1) in some sense (mild solutions) to the abstract ODE in Z given by

$$\dot{z}(t) = \mathcal{A}z(t) + (Bu(t),0)$$

with initial conditions $z(0) = z_0 = (\eta,\phi)$. More precisely, taking x as the solution to (1) on $[0,\infty)$ for a given (η,ϕ) and $u \equiv 0$ we define the homogeneous solution semigroup $\{S(t):t \geq 0\}$ by $S(t)(\eta,\phi) = (x(t;\eta,\phi), x_t(\eta,\phi))$. Then $\{S(t):t \geq 0\}$ is a $\mathcal{C}_0$ semigroup of bounded linear operators defined on Z with infinitesimal generator $\mathcal{A}$ defined on $\mathcal{D}(\mathcal{A}) = ((\phi(0),\phi): \phi \in W^n_{1,2}(-r,0))$ by $\mathcal{A}(\phi(0),\phi) = (L\phi,\dot{\phi})$ (see [3], [7]).

In the light of the discussion above, we are able to obtain solutions to the PID governed by the delay system (1) via solutions to the PID formulated in the Hilbert space Z given by

($\mathscr{P}$) Given input $u \in PC^n(0,T)$ and observation $\xi \in C^n(0,T)$, minimize

$$J(\gamma) = J(q,z_0) = |\hat{cz}(0;\gamma) + du(0) - \xi(0)|^2_{W_1} +$$

$$|\hat{cz}(T;\gamma) + du(T) - \xi(T)|^2_{W_2} + \int_0^T |\hat{cz}(s;\gamma) + du(s) - \xi(s)|^2_{W_3} ds$$

over $\Gamma = \mathscr{Q} \times \mathscr{S}$ subject to

$$\dot{z}(t) = \mathscr{A}(q)z(t) + (B(q)u(t),0)$$

$$z(0) = z_0.$$

Approximate solutions to the PID ($\mathscr{P}$) are obtained via the following two techniques, one involving discretization in the state only, the other involving simultaneous discretization in state and time.

Technique 1.

Choose a sequence of finite dimensional approximating subspaces Z^N of Z and let P^N represent the orthogonal projection

of Z onto Z^N along $(Z^N)^\perp$. Define the operators $\mathscr{A}^N(q): Z^N \to Z^N$ in such a way that they appropriately approximate the operator $\mathscr{A}(q)$ on Z for each $q \in \mathscr{Q}$ (see [3], [7]) and consider the sequence of approximating PID given by

$(\mathscr{AP})^1_N$: Given input $u \in PC^n(0,T)$ and observation $\xi \in C^n(0,T)$ minimize:

$$J^N(\gamma^N) = J^N(q, z_0^N) = |\hat{c}z^N(0;\gamma^N) + du(0) - \xi(0)|^2_{w_1} +$$

$$|\hat{c}z^N(T;\gamma^N) + du(T) - \xi(T)|^2_{w_2} + \int_0^T |\hat{c}z^N(s;\gamma^N) + du(s) - \xi(s)|^2_{w_3} ds$$

over $\Gamma^N = \mathscr{Q} \times P^N\mathscr{S}$ subject to

$$\dot{z}^N(t) = \mathscr{A}^N(q) z^N(t) + P^N(B(q)u(t), 0)$$

$$z^N(0) = z_0^N.$$

For each N, $(\mathscr{AP})^1_N$ represents a PID with finite dimensional state constraint. Standard gradient projection and conjugate gradient minimization techniques for optimization problems governed by ODE state equations (see [10]) may be employed to obtain solutions.

Technique 2.

Choose a sequence of finite dimensional approximating spaces Z_N (as opposed to subspaces as was the case in Technique 1) with

projection-like mappings $\pi_N: Z \to Z_N$. Define operators $\mathscr{A}_N(q): Z_N \to Z_N$ which in some sense approximate $\mathscr{A}(q)$ for each $q \in \mathscr{Q}$ (see [9]). Let $C(z)$ and $D(z)$ be rational function approximations to the exponential (RFAE) e^z, and consider the following sequence of approximating PID:

$(\mathscr{AP})_N^2$ Given input $\{u_N^j\}_{j=0}^{\rho N} \in \bigtimes_0^{\rho N} R^m$ and observation

$$\{\xi_N^j\}_{j=0}^{\rho N} \in \bigtimes_0^{\rho N} R^n \left(u_N^j = u(\tfrac{jr}{N}),\ \xi_N^j = \xi(\tfrac{jr}{N}),\ j = 0,1,2,\ldots\rho N, \right.$$

$$\left. \text{where } \rho \text{ is that positive integer for which } \rho r \le T < (\rho+1)r \right) \text{ minimize:}$$

$$J_N(\gamma_N) = J_N(q, z_{0N}) = |\hat{c}_N z_N^0(\gamma_N) + du_N^0 - \xi_N^0|_{w_1}^2 +$$

$$|\hat{c}_N z_N^{\rho N}(\gamma_N) + du_N^{\rho N} - \xi_N^{\rho N}|_{w_2}^2 + \frac{r}{N} \sum_{j=0}^{\rho N-1} |\hat{c}_N z_N^j(\gamma_N) + du_N^j - \xi_N^j|_{w_3}^2$$

over $\Gamma_N = \mathscr{Q} \times \pi_N \mathscr{S}$ subject to

$$z_N^{k+1} = C(\tfrac{r}{N}\mathscr{A}_N(q)) z_N^k + \tfrac{r}{N} D(\tfrac{r}{N}\mathscr{A}_N(q)) B_N(q) u_N^k, \quad k = 0,1,2\ldots\rho N-1$$

$$z_N^0 = z_{0N}$$

where $B_N(q)\eta \equiv \pi_N(B(q)\eta, 0)$ for $\eta \in R^n$.

For each N, $(\mathscr{AP})_N^2$ is a PID governed by a difference equation

and can be solved via standard numerical methods which are readily available (see [10]). Under appropriate conditions on Z^N, P^N, $\mathscr{A}^N(q)$, in Technique 1 (see [5]) and on Z_N, π_N, $\mathscr{A}_N(q)$, $C(z)$, $D(z)$ in the case of Technique 2 (see [9]), the following results may be established.

Theorem 1. Suppose $\{\bar{\gamma}_N\}$ (respectively $\{\bar{\gamma}^N\}$) is a sequence of solutions to the approximate problems $(\mathscr{A}\mathscr{P})_N^2$ (respectively $(\mathscr{A}\mathscr{P})_N^1$). Then there exists a $\bar{\gamma} = (\bar{q},\bar{z}_0) \in \Gamma$ and subsequences $\{\bar{\gamma}_{N_k}\} = \{(\bar{q}_{N_k},\bar{z}_{0N_k})\}$ and $\{\bar{\gamma}^{N_k}\} = \{(\bar{q}^{N_k},\bar{z}_0^{N_k})\}$ such that

(a) $\bar{q}_{N_k} \to \bar{q}$ in R^K $(\bar{q}^{N_k} \to \bar{q})$

and

(b) $\pi_{N_k}^{\dagger}\, \bar{z}_{0N_k} \to \bar{z}_0$ in Z $(\bar{z}_0^{N_k} \to \bar{z}_0$ in $Z)$

where $\pi_{N_k}^{\dagger} : Z_{N_k} \to Z$ denotes the Moore-Penrose generalized inverse of π_{N_k} (see [8]). Moreover, in both cases $\bar{\gamma}$ will be a solution to problem $(\mathscr{P})$. If the solution to problem $(\mathscr{P})$ is unique, then the sequences $\{\bar{\gamma}_N\}$, $\{\bar{\gamma}^N\}$ themselves converge to $\bar{\gamma}$ in the above sense.

In both Techniques 1 and 2, schemes satisfying the conditions required to establish the veracity of the above theorem may be realized via the construction of $Z^N, P^N, \mathscr{A}^N(q)$ or Z_N, π_N, $\mathscr{A}_N(q)$ based upon finite difference or spline approximations (see [3],[7],[9]).

The RFAE $C(z)$ and $D(z)$ can be chosen from among a subclass of the Padé approximations to e^z (see [9]). Proofs of the convergence stated in the theorem rely heavily upon results from functional analysis and linear semigroup theory.

We have tested the above techniques on a number of examples (see [4] for extensive tests of Techique 1) using the "averaging" and "spline" state approximations of [3] and [7] respectively. We present here a typical example to illustrate and compare results for the two techniques.

Example

We consider the scalar equation

$$\dot{x}(t) = a_0 x(t) + a_1 x(t-1) + u(t)$$

with step input $u = \chi_{[.1,\infty)}$, initial data $x(\theta) = 1$, $-1 \leq \theta \leq 0$, and observations $c(t) = x(t)$. "Data" ξ was generated on $[0,2]$ by integrating the equation exactly with true parameter values $a_0^* = .05$ and $a_1^* = -4.0$. (The techniques were also tested with "data" generated by adding random noise to the true solution. The resulting parameter estimates obtained are essentially unchanged from those obtained using "data" without noise.) For several values of approximation level N, iterative techniques with startup values $a_0^{N,0} = .03$, $a_1^{N,0} = -3.0$ were used to solve the approximating problems $(\mathscr{AP})_N^1$ and $(\mathscr{AP})_N^2$ corresponding to this "data" ξ on $[0,2]$. The results obtained are presented in tabular form below. In both techniques the AVE refers to the "averaging" state approximation of [3] while SPL 1 refers to the

piecewise linear spline state approximations of [7]. In Technique 2, we used the Padé RFAE

$$C(z) = P_{22}(z) = (1 + \tfrac{1}{2} z + z^2/12)/(1 - \tfrac{1}{2} z + z^2/12)$$

and

$$D(z) = P_{20}(\tfrac{1}{2} z) = (1 - \tfrac{1}{2} z + z^2/8)^{-1}$$

which correspond to time discretization schemes of approximation index $q = 4$ (see [9] for details and further discussions).

TECHNIQUE 1

	AVE		SPL 1	
N	$\bar{a}_0^N$	$\bar{a}_1^N$	$\bar{a}_0^N$	$\bar{a}_1^N$
2	1.0869	-4.6236	.0995	-4.1639
4	.6525	-4.3160	.0417	-4.0523
8	.3825	-4.1660	.0439	-4.0222
16	.2245	-4.0898	.0449	-4.0151
32	.1384	-4.0505	.0454	-4.0133
TRUE VALUES	.0500	-4.0000	.0500	-4.0000

TECHNIQUE 2

	AVE		SPL 1	
N	$\bar{a}_{0N}$	$\bar{a}_{1N}$	$\bar{a}_{0N}$	$\bar{a}_{1N}$
2	1.1742	-4.9316	.1564	-4.3972
4	.6325	-4.4671	.0045	-4.1682
8	.3368	-4.2838	-.0121	-4.1307
16	.2104	-4.1252	.0292	-4.0488
32	.1400	-4.0460	.0474	-4.0089
TRUE VALUES	.0500	-4.0000	.0500	-4.0000

The results obtained from our comparisons of Techniques 1 and 2 on a number of examples reveal that when the approximations reprented by $C(\frac{r}{N}\mathscr{A}_N)$ and $D(\frac{r}{N}\mathscr{A}_N)$ in $(\mathscr{A}\mathscr{P})_N^2$ are of an order comparable to that of the scheme used in solving the ordinary differential equat ion for z^N in $(\mathscr{A}\mathscr{P})_N^1$, the parameter estimates obtained are comparable. The implementation of Technique 2, which involves a simple difference equation approximation for the delay equation, is, in many cases, quite simple. For both techniques our studies indicate that the first order spline methods of [7] lead to schemes that are superior in many situations to those based on the averaging methods of [3] (see [4] and [9]). Of course, when one employs the cubic spline approximations of [7] (e.g., see [9]) even more impressive results are obtained.

The techniques discussed in this paper have also been used with success for parameter identification in problems where the delays $\tau_1, \tau_2, \ldots, \tau_\nu = r$ themselves are among the parameters to be estimated (see [1], [4], [6] for Technique 1; numerical results for Technique 2 in this case along with proofs will appear in a forthcoming manuscript) even in situations where $\frac{\partial x}{\partial \tau_i}$ does not exist (for discussions of this and its significance, see [1]). Finally, the methods are also applicable to certain classes of nonlinear parameter estimation and control problems (see [2], [6], [9]).

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